Special Relativity:
Time Dilation and the Speed of Light



- GPS drifts about 11 km/day without relativistic clock corrections
- Medical electron linacs need $\gamma \approx 43$ to steer beams correctly
- Two postulates force revisions of simultaneity, time, and velocity addition
- Goals: distinguish proper time from coordinate time, compute γ, read light-clock diagrams
- Roadmap: postulates → simultaneity → light clock → evidence → twin paradox → limits



Why Relativity Is an Everyday Engineering Fact

- GPS clocks: −7 μs/day from speed, +45 μs/day from weaker gravity, net +38 μs/day
- Uncorrected GPS positions would drift roughly 10–11 km per day
- First satellites carried an on/off switch; it had to stay on
- Hospital electron linacs need $\gamma \approx 43$ to steer 22 MeV beams correctly
- At low speeds, relativity reduces to Newton—why we don't notice it
- Framing question: if light's speed is the same for everyone, what gives—space, time, or both?

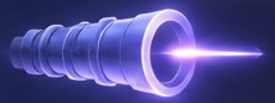


Clock correction:
+38 μs/day

Position error:
10–11 km/day



μs/day



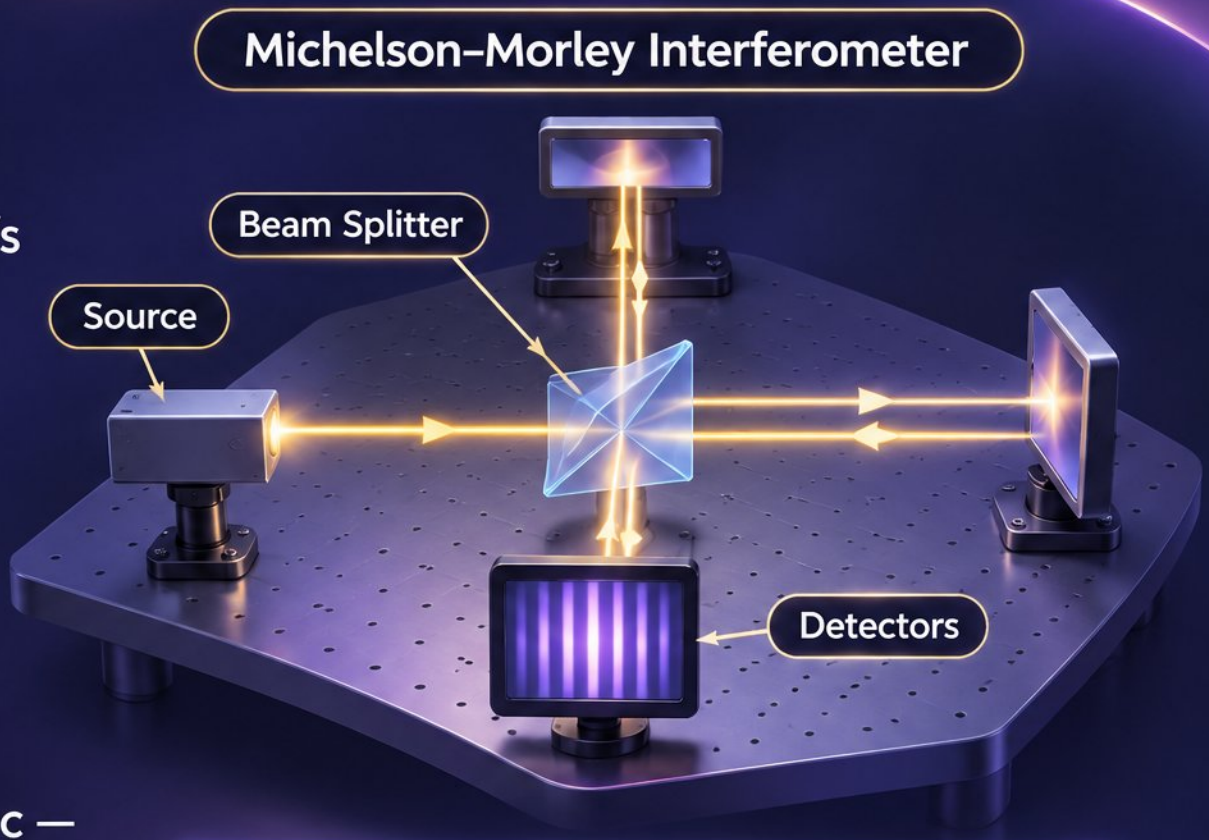
$\gamma \approx 43$

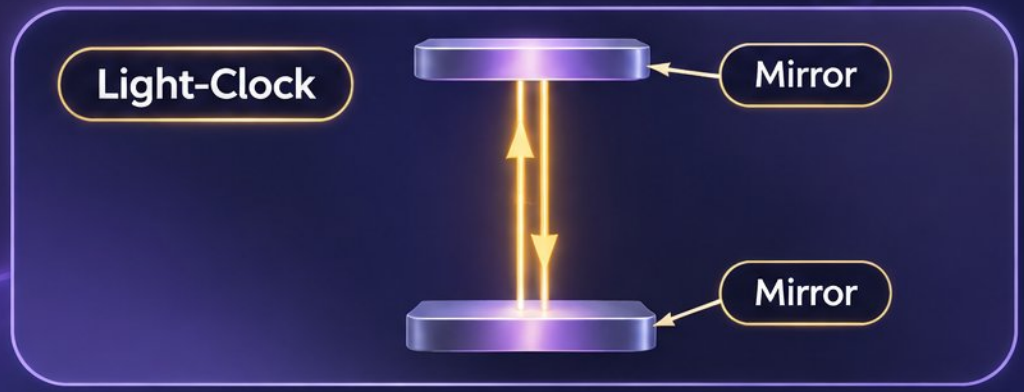


22 MeV

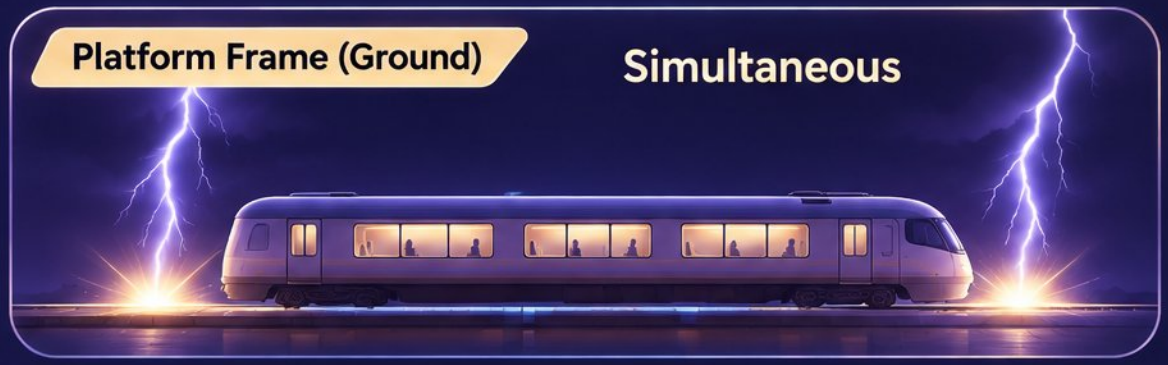
Two Postulates and the Constant Speed of Light

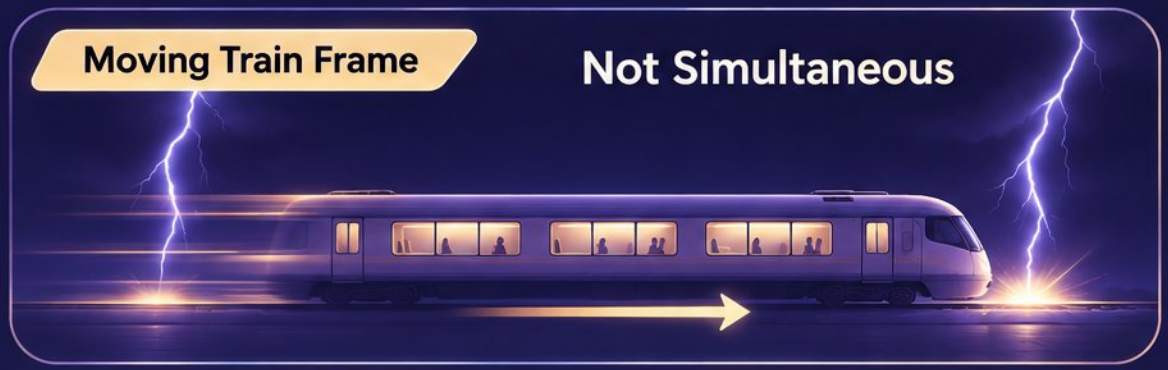
- Postulate 1: laws of physics are identical n every inertial frame
- Postulate 2: c = 299,792,458 m/s in all inertial frames, source motion irrelevant
- Michelson–Morley found no ether drift — the most famous failed experiment
- You can never catch up to light; no observer sees frozen light
- Velocities don't simply add near c — Newtonian kinematics must yield
- Misconception alert: c is exact only in vacuum; light slows in glass, water, air





Relativity of Simultaneity: The First Surprise







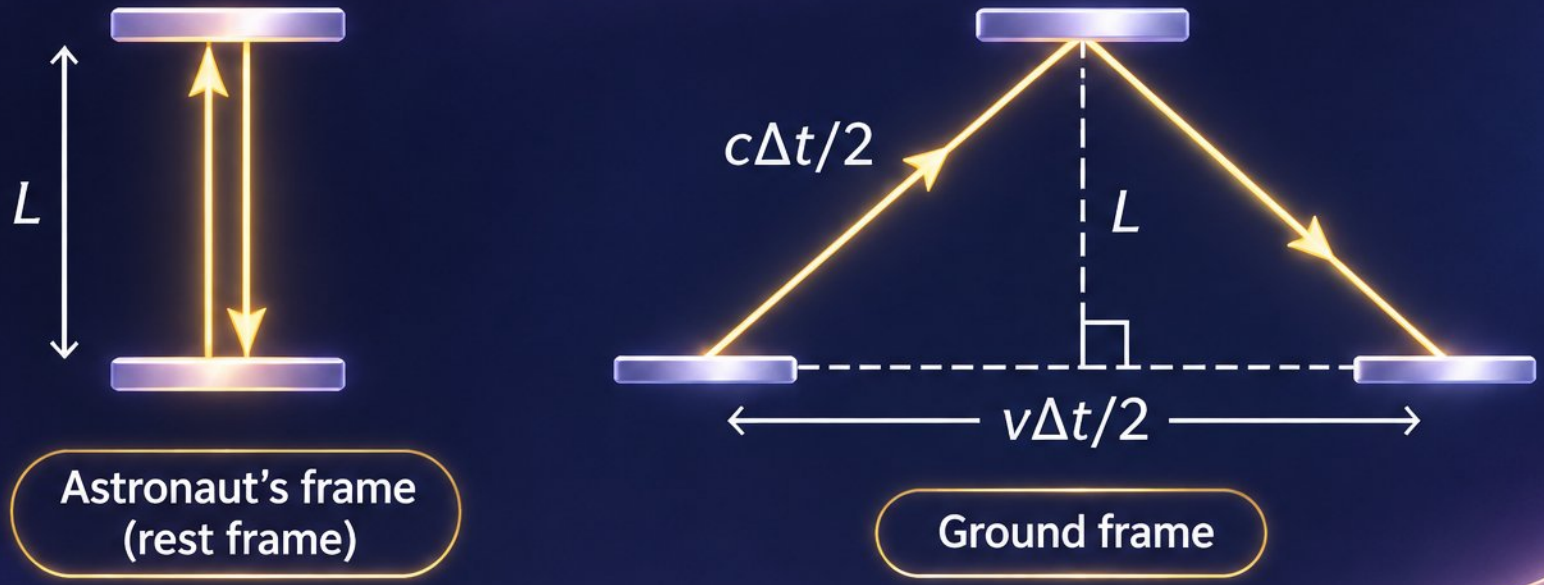
At Rest



In Motion

- Clocks sync by exchanging light signals → "same time" is a procedure
- Train-platform lightning: simultaneous on ground, not in moving train
- Not an illusion: each frame is equally valid; no privileged frame
- Time-like pairs keep causal order; space-like pairs can reverse
- Lost absolute simultaneity is the conceptual root of time dilation

The Light Clock: Deriving the Lorentz Factor



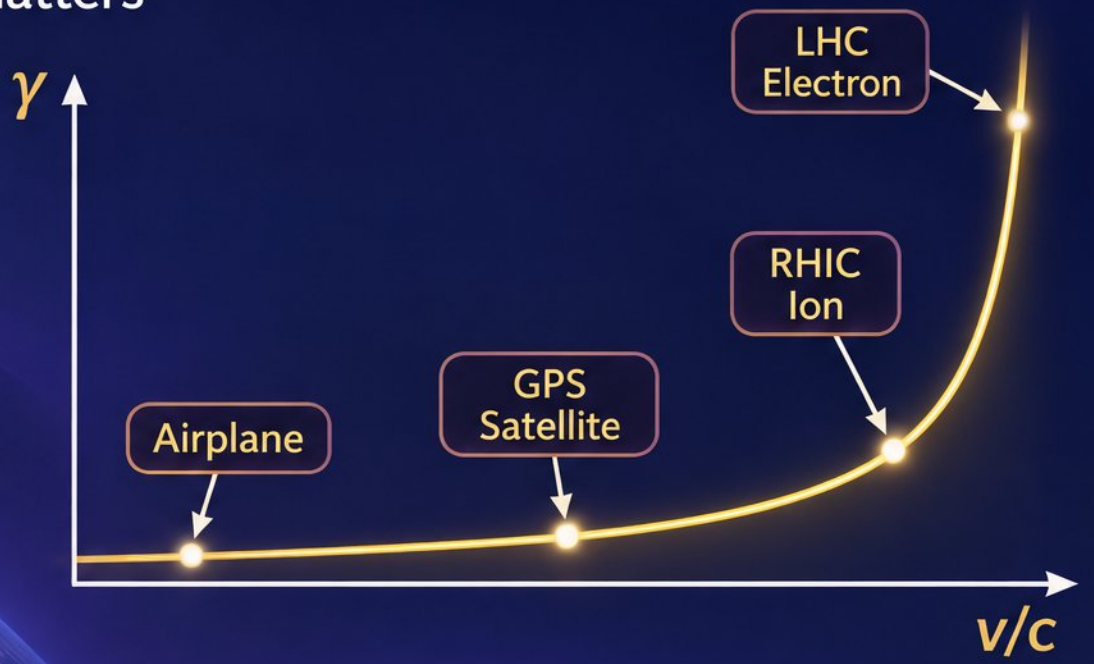
- Photon bounces between two mirrors, distance L apart
- One tick = round trip; astronaut measures $\Delta t_0 = 2L/c$
- Ground observer sees a longer, oblique light path
- Pythagoras gives $(c\Delta t/2)^2 = L^2 + (v\Delta t/2)^2$
- Result: $\Delta t = \gamma \Delta t_0$, with $\gamma = 1/\sqrt{(1 - v^2/c^2)}$
- $\gamma - 1 \approx 10^{-13}$ for an airliner; $\gamma \approx 7.1$ at $0.99c$

Working with γ: A Table of Speeds and Ages

- Effects measurable only as v/c approaches 1
- Table: 0.5c → γ = 1.15; 0.9c → 2.29; 0.99c → 7.09
- Slow rule of thumb: $\gamma - 1 \approx v^2/2c^2$
- Jet airliner: 10^{-13}; Earth's orbit: ~10^{-8}
- Traveler ages $\tau = t/\gamma$; only v/c matters

v/c	γ
0.5	1.15
0.9	2.29
0.99	7.09
0.997	12.92
near c	very large

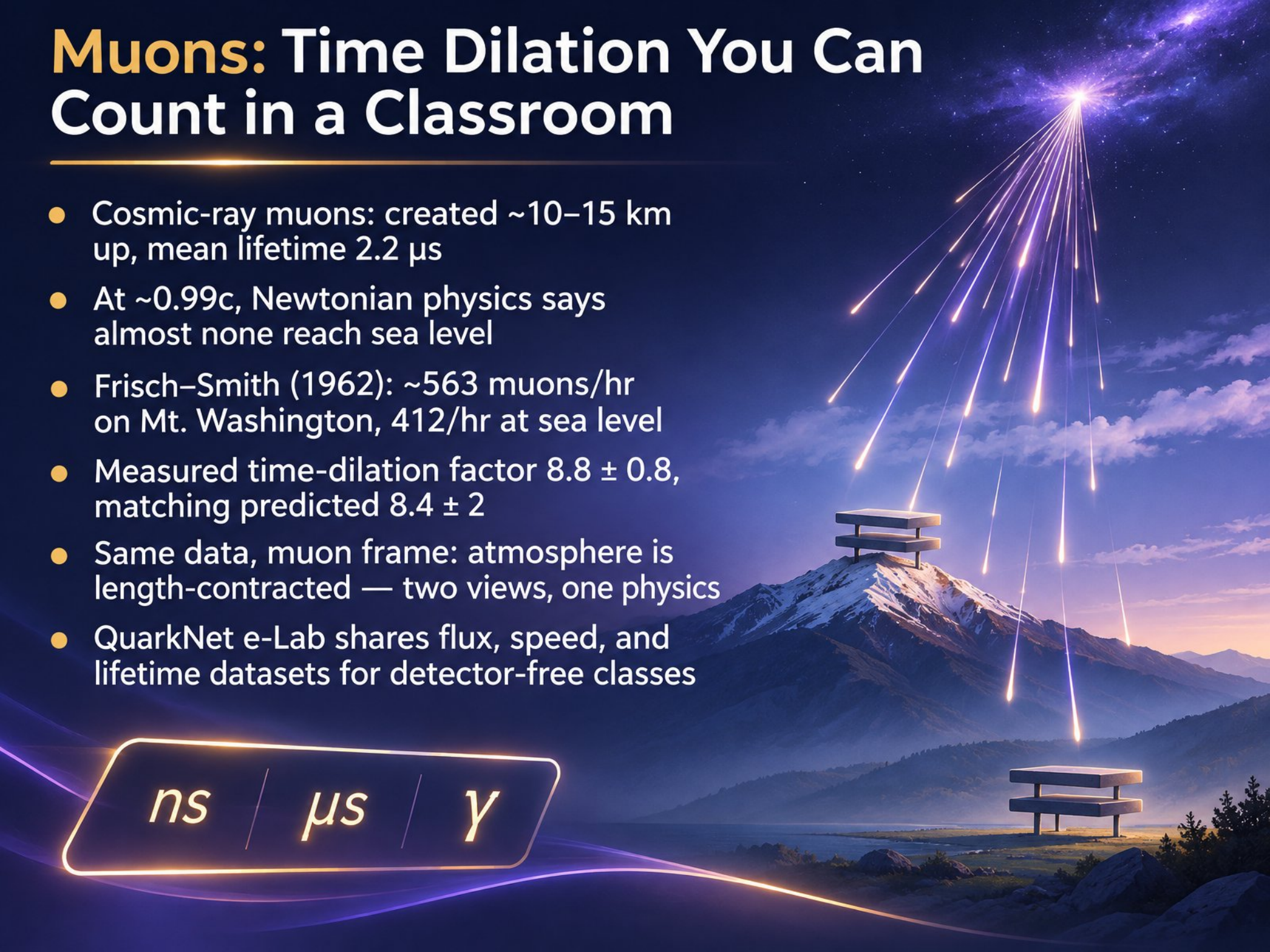




Muons: Time Dilation You Can Count in a Classroom

- Cosmic-ray muons: created ~10–15 km up, mean lifetime 2.2 μs
- At ~0.99c, Newtonian physics says almost none reach sea level
- Frisch–Smith (1962): ~563 muons/hr on Mt. Washington, 412/hr at sea level
- Measured time-dilation factor 8.8 ± 0.8, matching predicted 8.4 ± 2
- Same data, muon frame: atmosphere is length-contracted — two views, one physics
- QuarkNet e-Lab shares flux, speed, and lifetime datasets for detector-free classes

ns | *μs* | *γ*



Clocks in Flight: From Jet Aircraft to GPS

- Hafele–Keating (1971): eastward clocks lost 59 ns, westward gained 273 ns
- Directional asymmetry requires both special and general relativity
- 1996 NPL flight: 39 ± 2 ns measured versus 39.8 ns predicted
- GPS clocks: −7 μs/day (SR) + 45 μs/day (GR) = +38 μs/day net
- Satellite clocks detuned to 10.22999999543 MHz before launch
- Optical clocks now resolve gravitational time dilation across millimeters











Hafele–Keating (1971)

Measured: −59 ns / +273 ns

Predicted: −59 ns / +273 ns

1996 NPL flight

Measured: 39 ± 2 ns

Predicted: 39.8 ns

2010 round-the-world

Measured: −7 ns

Predicted: −7 ns

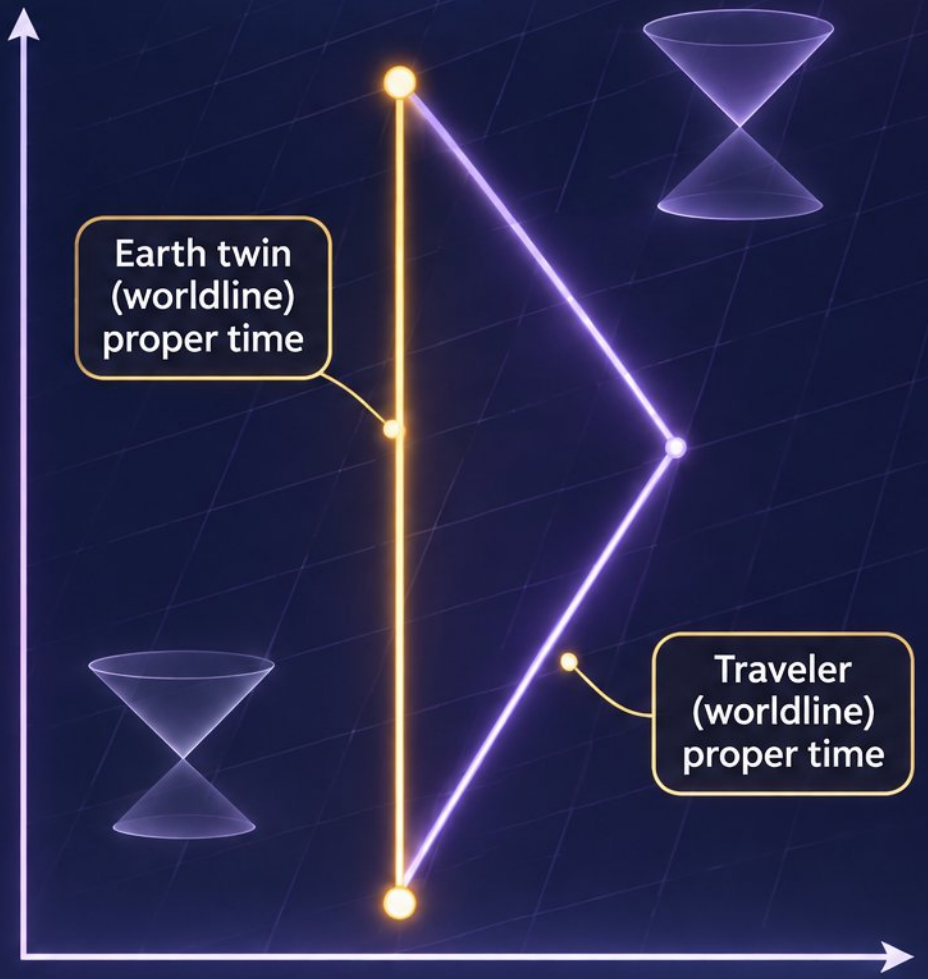
Optical-lattice clocks

Measured: across millimeters

Predicted: across millimeters

The Twin Paradox: Why Acceleration Breaks the Symmetry

- Setup: one twin stays on Earth, the other travels out and back
- Naive expectation: each sees the other's clock run slow — a contradiction
- Resolution: the traveler changes inertial frames at turnaround, the stay-at-home does not
- Spacetime diagram: proper time is the length along each worldline
- The inertial path between two events has the longest proper time
- Misconception: acceleration does not "slow the clock" directly; frame changes and proper time do
- Symmetry restored: compare clocks only when twins reunite



Misconceptions and the Speed Limit

Common Myths?

- **Traveler perception?** → Time dilation: measured coordinate time vs. proper time, not perception



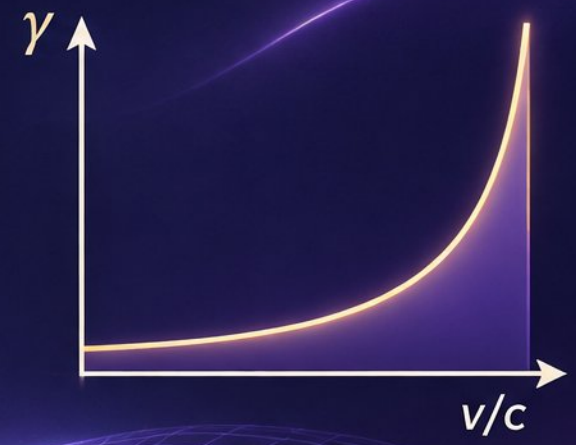
- **Velocity addition?** → Velocities don't add: $u = (u' + v)/(1 + u'v/c^2)$; light + anything = c



- **Reaching c?** → Kinetic energy γmc^2 diverges as $v \to c$; accelerators never reach c



- Time dilation is measured coordinate time vs. proper time, not perception
- Velocities don't add: $u = (u' + v)/(1 + u'v/c^2)$; light + anything = c
- Kinetic energy γmc^2 diverges as $v \to c$; accelerators never reach c
- Relativistic momentum is γmv; invariant mass stays fixed
- Beyond SR: acceleration and gravity also affect clocks (GPS, Hafele–Keating)



Open Questions and the Precision Frontier

- Atomic clocks tighten tests of Lorentz invariance year by year
- ACES launched to the ISS in 2025; plans a 10× more stringent relativity test
- NIST/CU mountaintop clock begins data collection in 2026
- Optical ion clocks probe quantum proper time — an active research front
- Settled: time dilation and c as the invariant speed limit
- Present these as live questions, not challenges to the theory



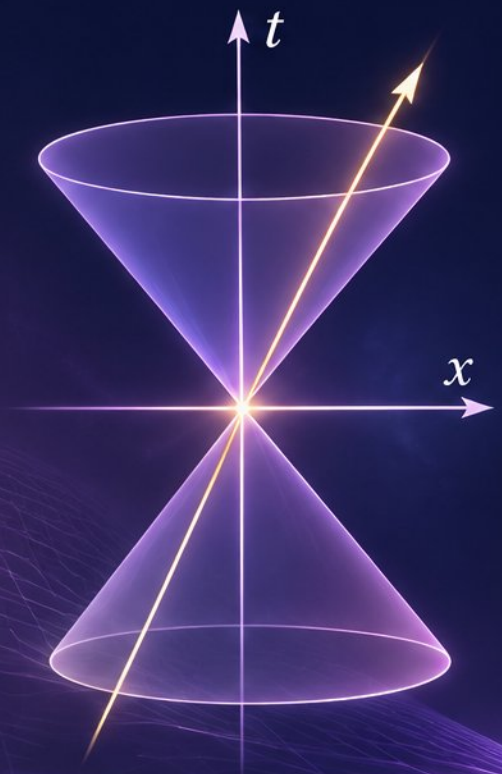
Classroom Toolkit: Demonstrations, Simulations, and Data

- 90-min light-clock simulation lesson: gains strongest with explicit light-postulate discussion
- Hands-on: QuarkNet muon datasets, TeachSpin apparatus, Frontiers virtual muon lab (ages 16–18)
- GPS activities: compare predicted +38 μs/day with observed clock corrections
- Question habit: "Who measures proper time here?" and "Which frame is this?"
- Assess concepts—simultaneity, proper time, misconception diagnosis—not just algebra



Audience Takeaways and Further Reading

- Adult learners: GPS uses time dilation every second — relativity is practical
- Students: light clock + Pythagoras gives γ; gateway to momentum and $E = mc^2$
- Educators: postulates → simultaneity → light clock → muons → twin paradox
- Sources: OpenStax Ch. 28/10, Feynman Vol. I Ch. 15, NIST, QuarkNet
- One invariant c makes time frame-dependent — measured in jets, muons, GPS







c





OpenStax Ch. 28/10

Feynman Vol. I Ch. 15

NIST

QuarkNet

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