

Introduction to Symmetry: Patterns and Transformations



- Symmetry means invariance: an object has symmetry if a move leaves it looking the same



- Three rigid motions preserve size and shape: reflection (flip), rotation (turn), translation (slide)



- Symmetry appears in nature, art, architecture, and everyday design



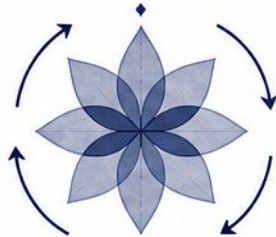
- Goal: identify invariant relationships — distances, angles, orientation, parallelism, and fixed points



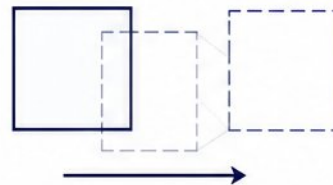
REFLECTION (FLIP)



ROTATION (TURN)

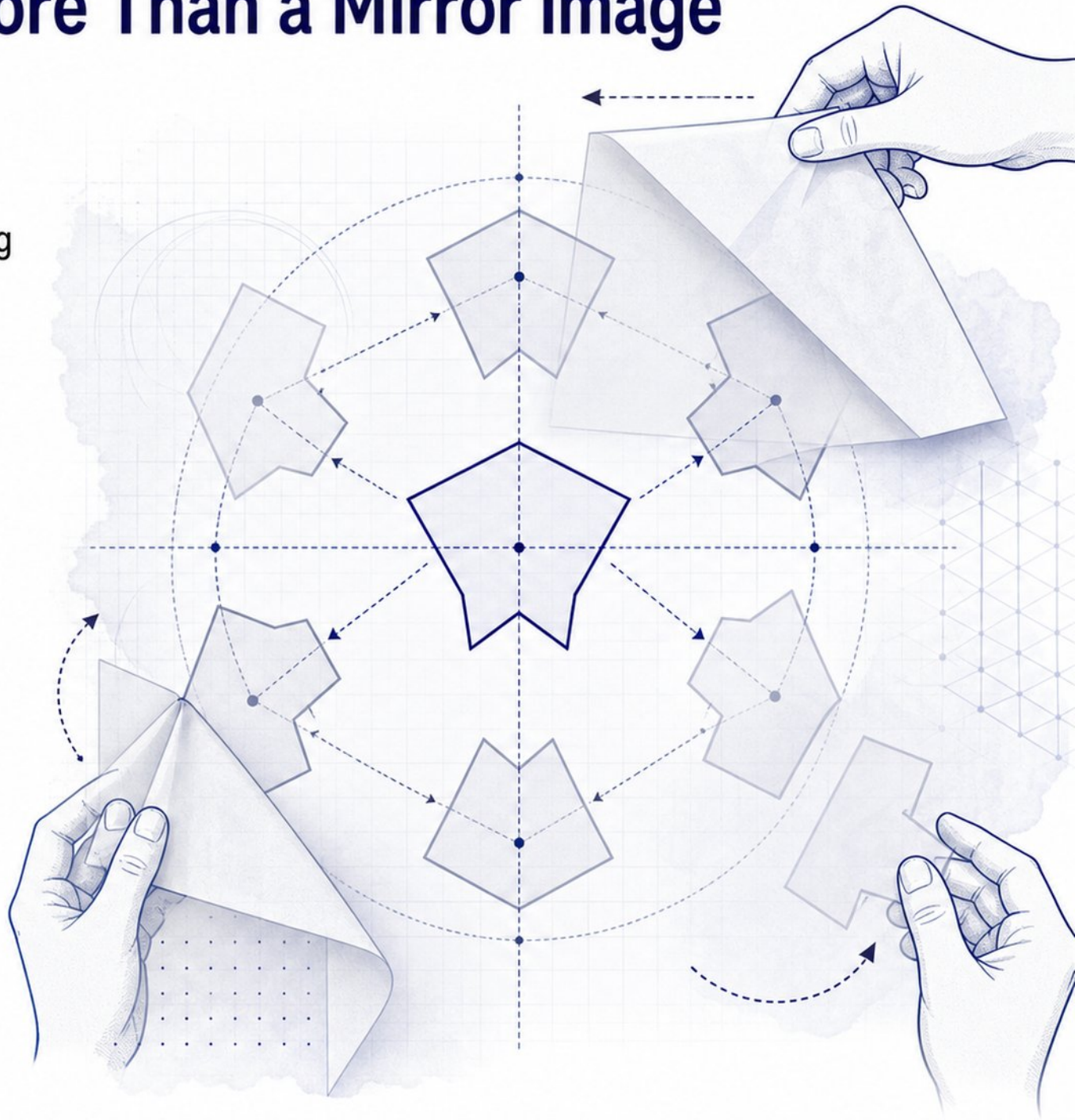


TRANSLATION (SLIDE)



Symmetry Is More Than a Mirror Image

- Symmetry defined as an isometry: a distance-preserving map that leaves the whole figure unchanged.
- Distinguish between a single shape and a repeated pattern formed by mapping it.
- Invariance means certain properties stay the same while others may change under transformation.
- Build intuition with everyday metaphors: tracing paper sliding, folding, and rotating.



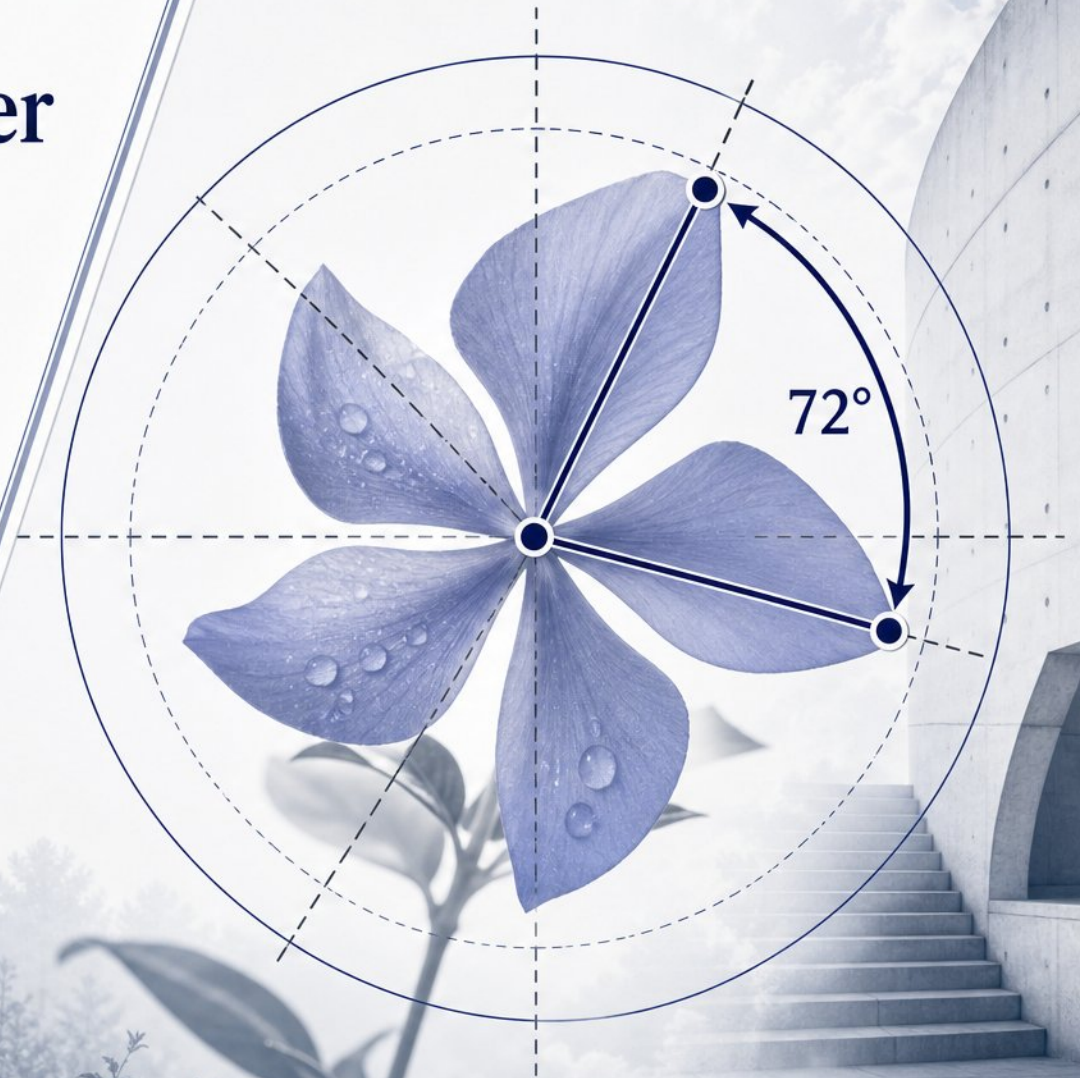
Reflection: The Flip Across a Mirror Line

- Every point maps to the opposite side at the same perpendicular distance
- The mirror line is the perpendicular bisector of any point-image segment
- Lengths, angles, parallelism, and collinearity are all preserved
- Orientation reverses; only points on the mirror line stay fixed

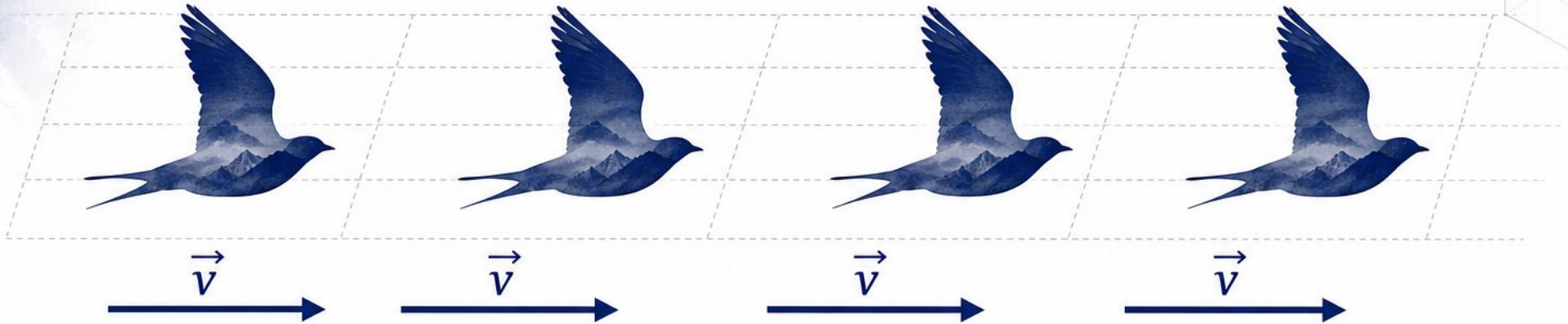


Rotation: Turning Around a Fixed Center

- - A turn around a fixed center point through a specified angle
- - Preserves distances, lengths, angles, and orientation (no flipping)
- - Find the center using perpendicular bisectors of point-to-image segments
- - Only the center point stays fixed during the rotation



Translation: A Slide Without Turning



- - A translation slides every point the same distance in the same direction, described by a vector
- - Corresponding pre-image and image segments remain equal in length and parallel
- - Invariants: all distances, angles, parallelism, and orientation stay unchanged
- - Key contrast: no fixed points—everything moves; the basis for frieze and wallpaper patterns

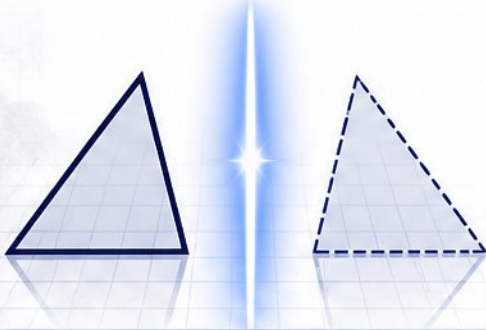
The Invariant & Changed Table

The Invariant & Changed Table	
Invariant (Unchanged)	Changed
 All distances	 Position of points
 All angles	 Locations of figures
 Parallelism	 Placement in the plane
 Orientation (no turning)	—

Comparing the Three Transformations

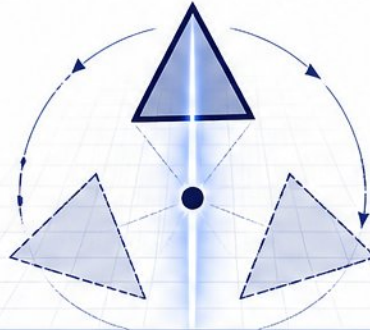
Glowing Axis of Transformation

REFLECTION



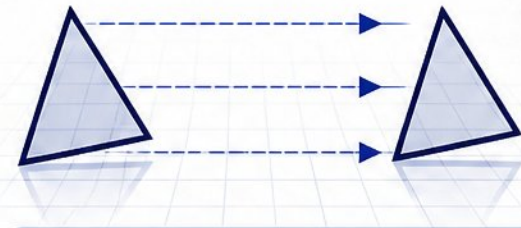
- Reflection; reverses orientation, fixes every point on the mirror line

ROTATION



- Rotation; preserves orientation, fixes exactly one point (the center)

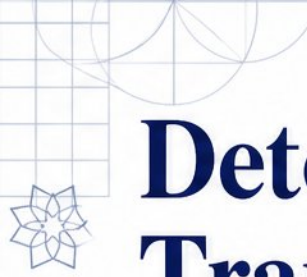
TRANSLATION



- Translation; preserves orientation, no fixed points, all slide-parallel lines stay



- Orientation, fixed points, and parallel lines reveal which transformation was used



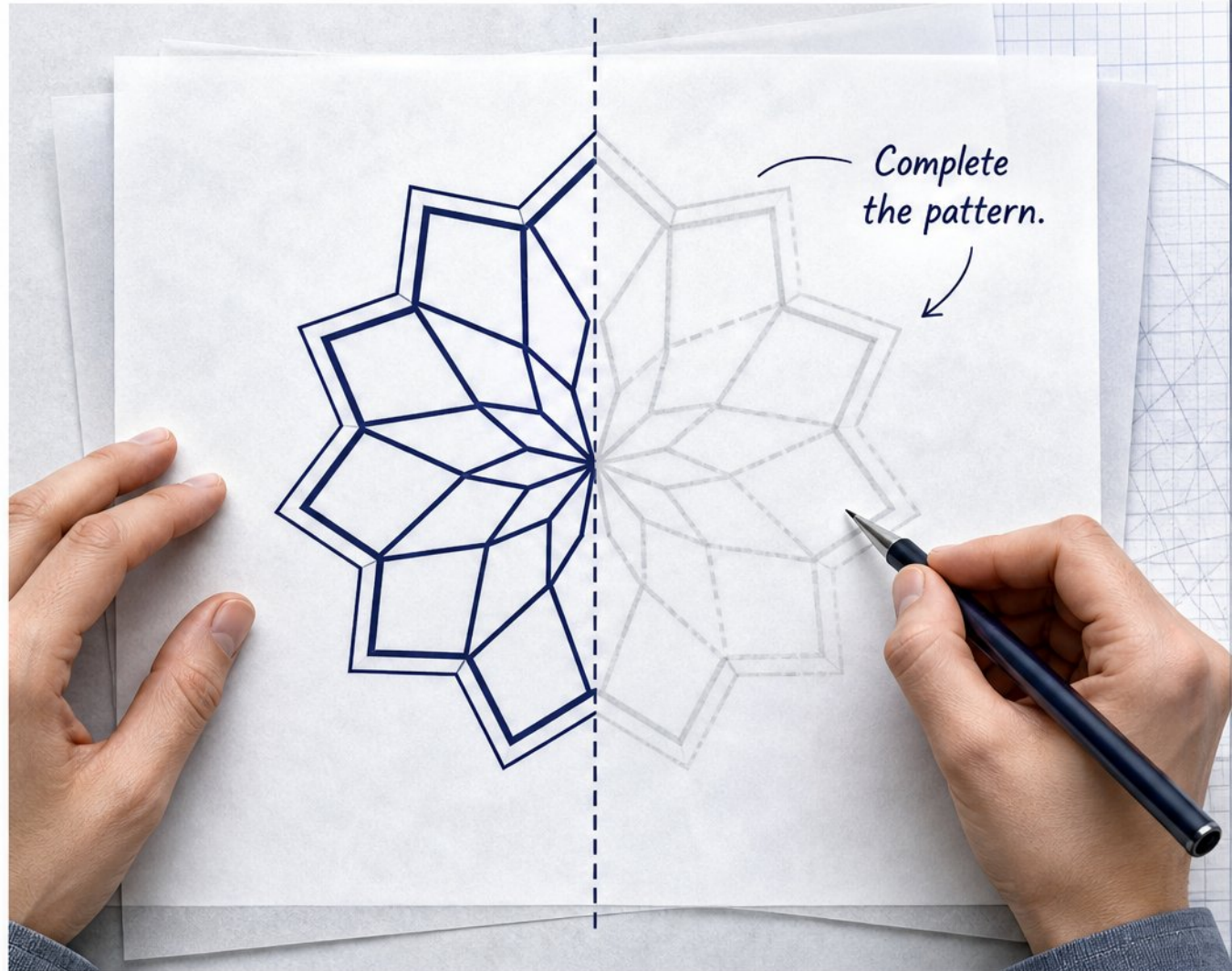
Detecting Transformations in Everyday Patterns

- ◆ Identify transformations by checking orientation, fixed points, and distance
- ◆ Expand beyond single shapes to frieze borders, rosettes, and tessellations
- ◆ A 180° rotation preserves orientation unlike a reflection
- ◆ Some complex patterns combine multiple transformations at once
- ◆ Practice with real designs from art, architecture, and nature

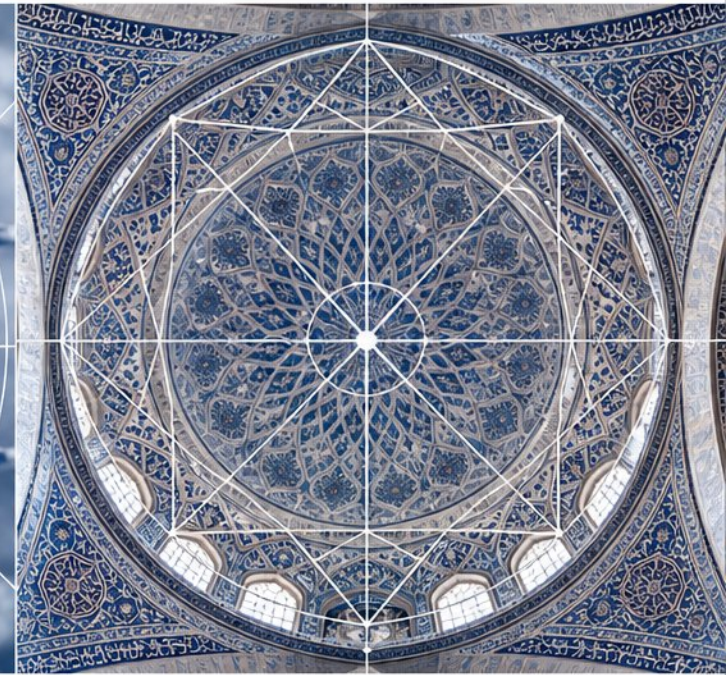
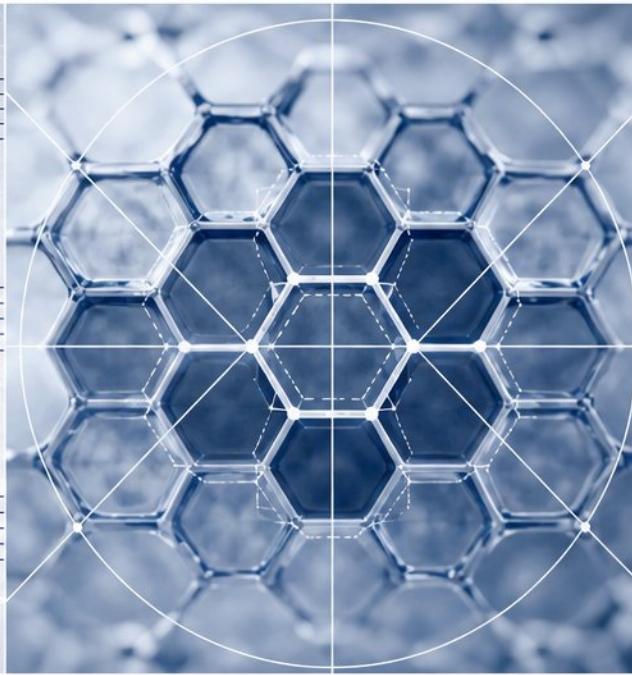


Using Symmetry to Complete and Predict

- Complete a design when given a mirror line, center, or translation vector.
- Reflect a point, rotate a triangle, or translate a simple motif.
- Designers use symmetry for logos, facades, and textile repeats.
- Invariants like angles and distances guarantee the predicted shape is correct.



Symmetry in Art, Music, and Science



- **Music:** transposition (translation), retrograde (reflection), and inversion structure compositions



- **Chemistry & physics:** molecular symmetry and conservation laws rely on transformation invariance



- **Design & architecture:** translation, rotation, and reflection build visual balance



- **Invariance mindset:** a transferable skill to find what stays the same when something changes

Common Misconceptions and How to Correct Them



- “Balanced” doesn’t always mean a mirror line — a parallelogram looks balanced but cannot be folded onto itself



- A 180° rotation is not a reflection — orientation (handedness) stays the same

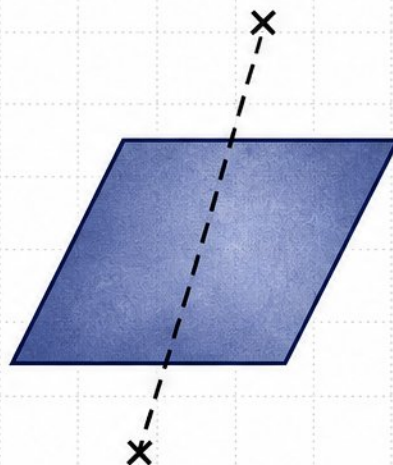


- A pure slide never flips, turns, or twists — orientation is fully preserved



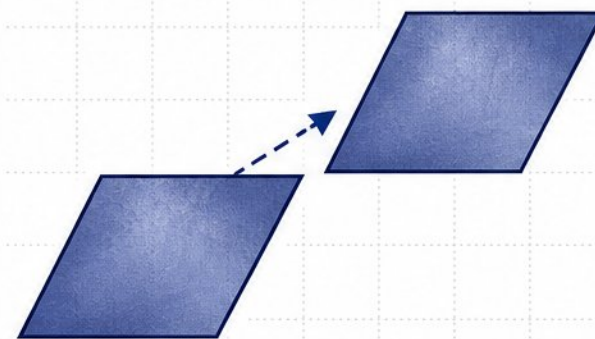
- Always check three clues: handedness, landmark path, and fixed points

NOT A LINE OF SYMMETRY



A dashed line cannot fold the parallelogram onto itself.

PURE SLIDE (TRANSLATION)



The shape slides along a straight path. No flip, turn, or twist occurs.

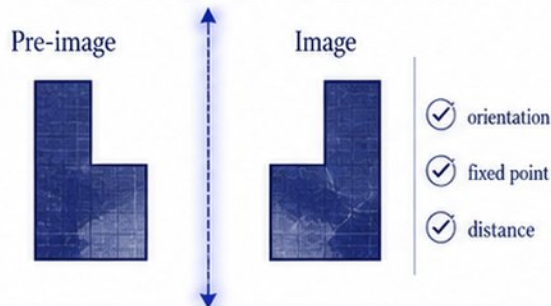
THE INVARIANT & CHANGED TABLE

INVARIANT (STAYS THE SAME)		CHANGED (DIFFERENT)	
	Handedness (orientation/ left-right order)		Would change under reflection (but not in a slide)
	Landmark path (the way a point moves)		Would reverse under reflection (but not in a slide)
	Fixed points (points that stay put)		Would change under other transformations (but not in a slide)

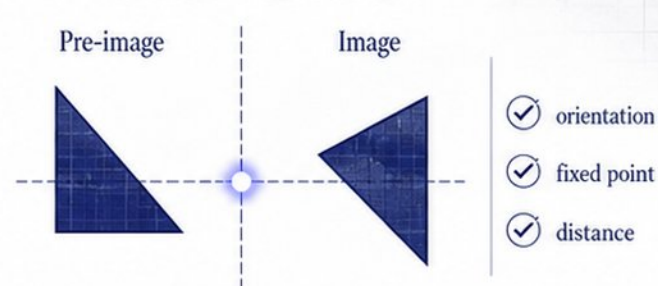
Hands-On Identification Practice

- Diagnose each pre-image/image pair: reflection, rotation, or translation.
- Watch for ambiguous cases where orientation tricks the eye.
- Apply the invariance test: check fixed points, orientation, and distances.
- Explain your reasoning aloud to strengthen understanding.

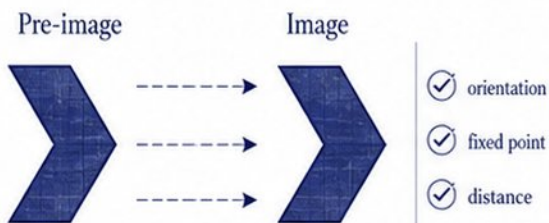
REFLECTION ACROSS A VERTICAL LINE



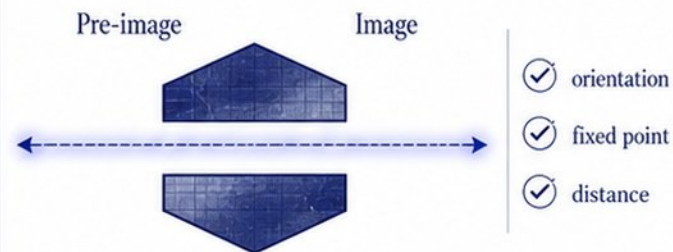
ROTATION ABOUT A POINT



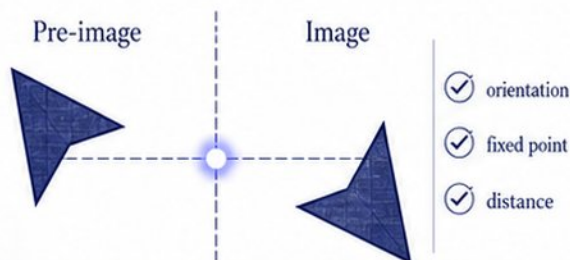
TRANSLATION TO THE RIGHT



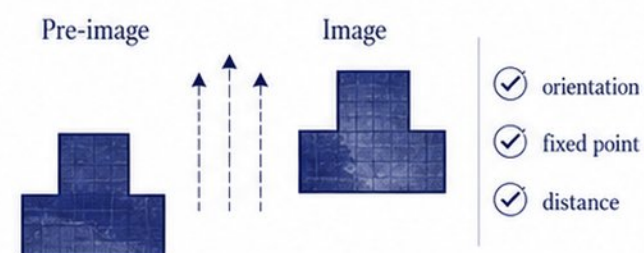
REFLECTION ACROSS A HORIZONTAL LINE



ROTATION ABOUT A POINT

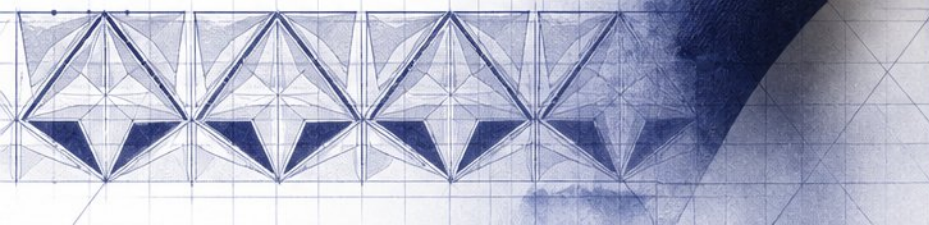
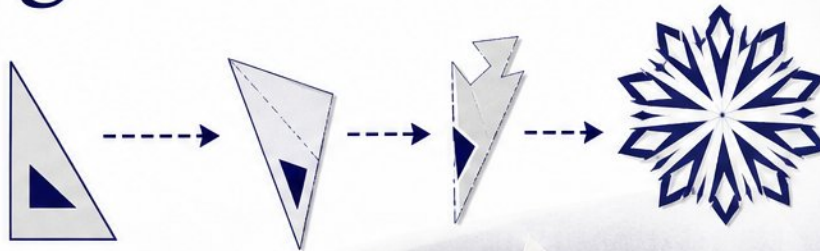


TRANSLATION UPWARD



Building Symmetric Designs

- ▶ Create patterns by reflecting, rotating, or translating a simple motif
- ▶ Use paper folding, tracing, or digital tools to form frieze patterns and rosettes
- ▶ Complete a partially built design using transformation rules
- ▶ Check consistency: What stays the same under each transformation?



Summary and the Invariance Mindset



- Symmetry = invariance under rigid transformations



- Reflection: mirror line, reversed orientation



- Rotation: fixed center, preserved orientation



- Translation: parallel slide, no fixed points



- Self-check: What stays the same? Which points are fixed?



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