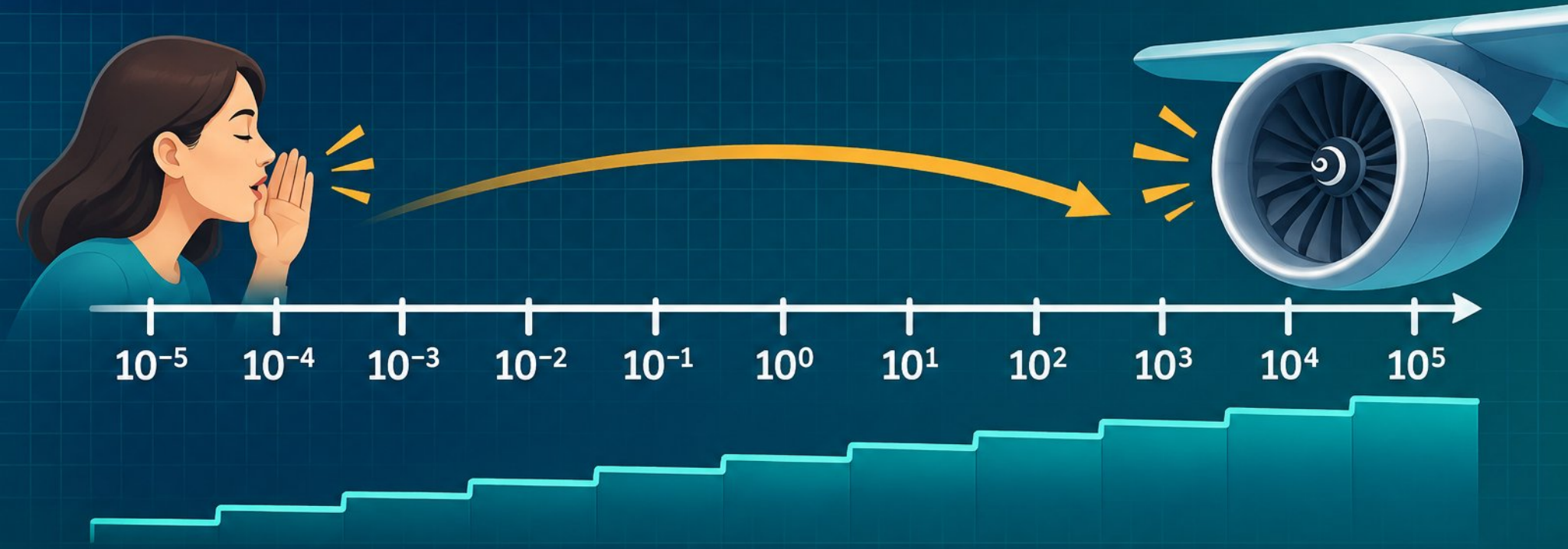
TS-0156 · Logarithmic Scales: pH, Decibels, and Earthquake Magnitude



- Three everyday sciences running on base-10 logs: pH, decibels, earthquake magnitude
- These quantities span 10+ orders of magnitude — linear intuition fails
- Goal: read a scale value, compare values as ratios, convert differences to factors
- Prerequisite: comfort with powers of ten (10^2, 10^-7); logs built from scratch
- Scope: conceptual understanding and arithmetic, not instrument calibration or research seismology

Why Some Quantities Need a Logarithmic Scale



- Powers of ten tame huge and tiny numbers: whisper vs jet engine ≈ 100 billion.
- Base-10 log = 'how many factors of ten': $\log_{10} 1000 = 3$, $\log_{10} 0.001 = -3$, $\log_{10} 1 = 0$.
- Log scale compresses wide ranges; equal steps mean equal ratios, not equal additions.
- Rule of thumb: +1 on the scale means ×10; +2 means ×100.
- No zero on a log scale; halfway between 10 and 100 is about 32, not 55.

Logarithm Refresher: Reading and Reversing Powers of Ten

- **Definition:** $\log_{10} x$ asks "10 to what power gives x?" → $10^{(\log_{10} x)} = x$
- **Laws of logs:** product → sum, quotient → difference, power → multiply exponent
- **Numbers below 1 give negative logs:** $\log_{10} 0.01 = -2$; log of 0 or negatives undefined
- **Subtracting two log readings gives a ratio:** difference d → ratio = 10^d
- **Master template:** every scale here is $L = c \cdot \log_{10}(Q/Q_0)$; c and Q_0 define each scale

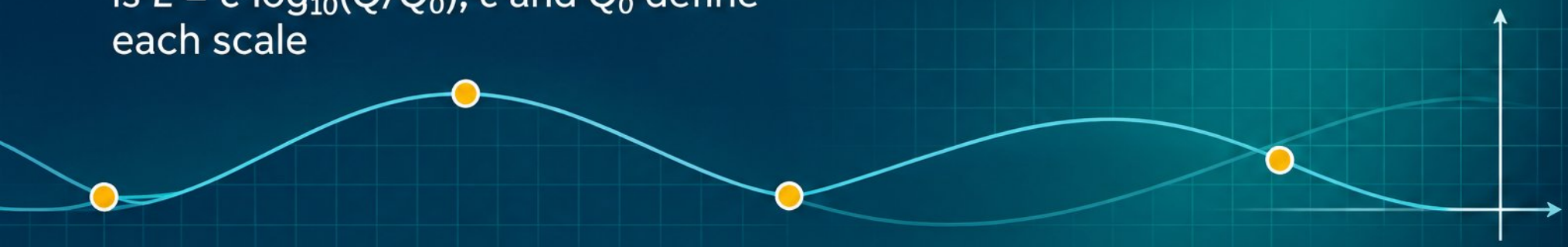
$$\log_{10}(10^k) = k$$

Product Rule

$$\log_{10}(ab) \leftrightarrow \log_{10} a + \log_{10} b$$

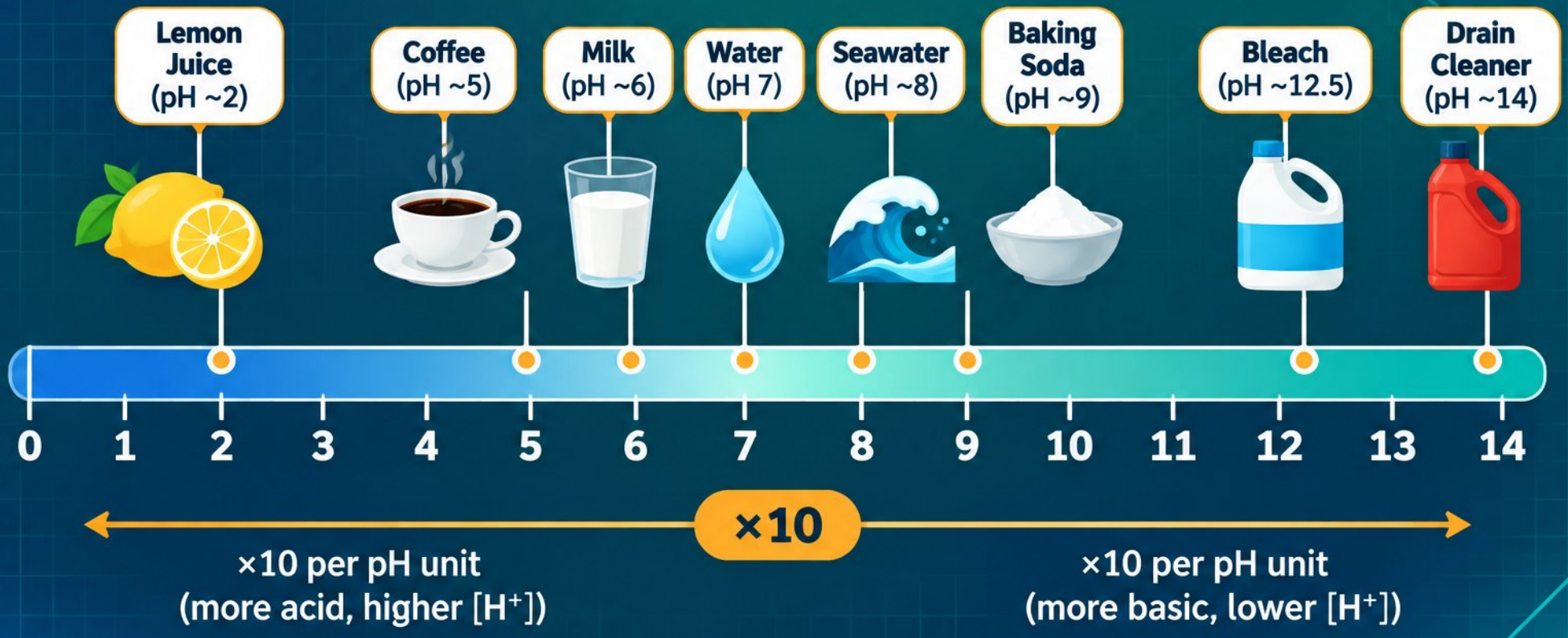
Quotient Rule

$$\log_{10}\left(\frac{a}{b}\right) \leftrightarrow \log_{10} a - \log_{10} b$$



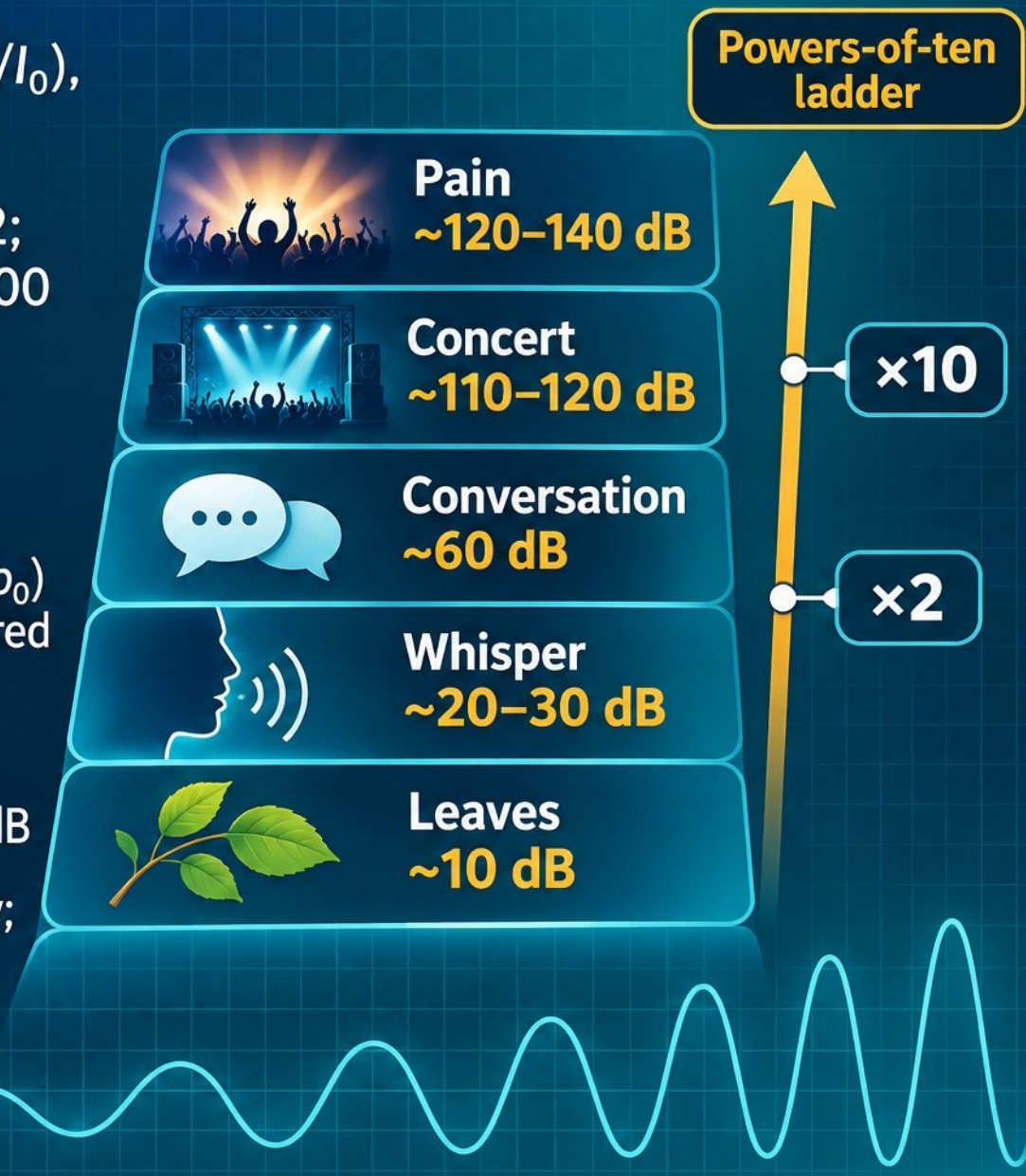
pH: A Logarithm of Hydrogen Ion Concentration

- pH = $-\log_{10}[H^+]$; $[H^+]$ in moles per liter
- Lower pH means more acid: pH 3 has **100×** the $[H^+]$ of pH 5
- Neutral is pH 7 at 25 °C; below 7 acidic, above 7 basic
- Typical values: lemon juice ~2, coffee ~5, seawater ~8, bleach ~12.5
- Invert with $[H^+] = 10^{(-pH)}$; the minus sign keeps readings positive



Decibels: Logarithms for Sound and Signal Intensity

- Sound intensity level: $L = 10 \cdot \log_{10}(I/I_0)$, $I_0 = 10^{-12}$ W/m² (hearing threshold)
- +10 dB = ×10 intensity; +3 dB ≈ ×2; +20 dB = ×100; +60 dB = ×1,000,000
- 0 dB means intensity equals the reference, not silence; negative dB values are possible
- Sound pressure level uses $20 \cdot \log_{10}(p/p_0)$ because power scales as pressure squared
- References: leaves ~10 dB · whisper ~20–30 dB · conversation ~60 dB · concert ~110–120 dB · pain ~120–140 dB
- The ear responds roughly logarithmically; +10 dB sounds about twice as loud

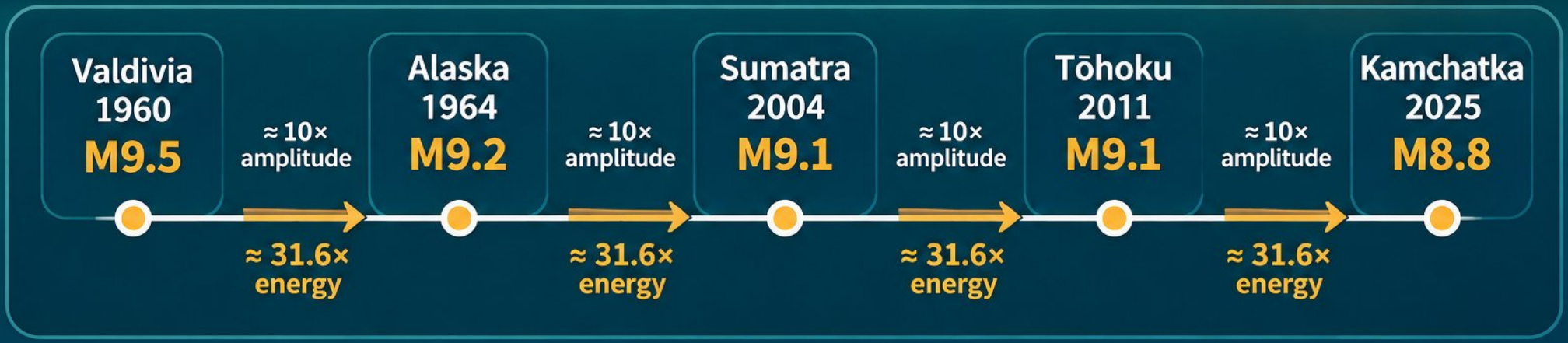


Earthquake Magnitude: Logarithms on a Massive Scale

- Magnitude = $\log_{10}$ of wave amplitude, plus distance and instrument corrections
- +1 magnitude ≈ **10×** amplitude, but ≈ **31.6×** energy ($10^{1.5}$)
- Richter M_L (1935): California-only, Wood-Anderson instrument, saturates for great quakes
- Moment magnitude $M_W = (2/3)\cdot\log_{10}(M_0) - 6.03$; reliable at all sizes
- M9.5 Chile 1960; M9.2 Alaska 1964; M9.1 Sumatra 2004, Tōhoku 2011; M8.8 Kamchatka July 2025
- USGS catalog: 100 events M8.0+ since 1900 (~1/year); 17 M8.5+ (~every 7–8 years)







Reading and Comparing Values Across the Three Scales

- Universal method: subtract readings, then raise 10 to the difference
- Ratio = 10^(difference) for pH and magnitude; 10^(difference/10) for dB



pH 3 vs pH 5

$5 - 3 = 2 \rightarrow 10^2$

pH 3 vs pH 5 → 2 units → 100× more acidic



70 dB vs 90 dB

$90 - 70 = 20 \rightarrow 10^2$

70 dB vs 90 dB → 20 dB → 10^2 = 100× the intensity



M5 vs M7

$7 - 5 = 2 \rightarrow 10^2$

M5 vs M7 → 100× amplitude; 10^(2×1.5) ≈ 1,000× energy

- Template: $L = c \cdot \log_{10}(Q/Q_0)$ gives $Q_2/Q_1 = 10^{(L_2 - L_1)/c}$; reference cancels
- Say answers as plain ratios: "100 times", "about 1,000 times"

Common Traps and How to Correct Them

Trap 1: M8 is not twice M4 — it is $10^4 = 10{,}000\times$ amplitude.

~~M8 = 2 × M4~~ → $10^4 = 10{,}000\times$

Trap 2: 20 dB = 10^(20/10) = 100×, not 10^{20}; divide by 10 first.

~~20 dB = 10^{20}~~ → 20 dB = 100×

Trap 3: Lower pH is more acidic, since pH = $-\log[H^+]$.

~~Higher pH = more acidic~~ → Lower pH = more acidic

Trap 4: pH 0 means $[H^+] = 1$ mol/L — strongly acidic, not acid-free.

~~pH 0 = acid-free~~ → $[H^+] = 1$ mol/L (strongly acidic)

Trap 5: Magnitude measures source size; shaking also depends on distance, depth, ground.

~~Magnitude = shaking size~~ → Magnitude = source size

Trap 6: Two 70 dB sources ≈ 73 dB, not 140 dB — dB values do not add.

~~Two 70 dB = 140 dB~~ → Two 70 dB ≈ 73 dB

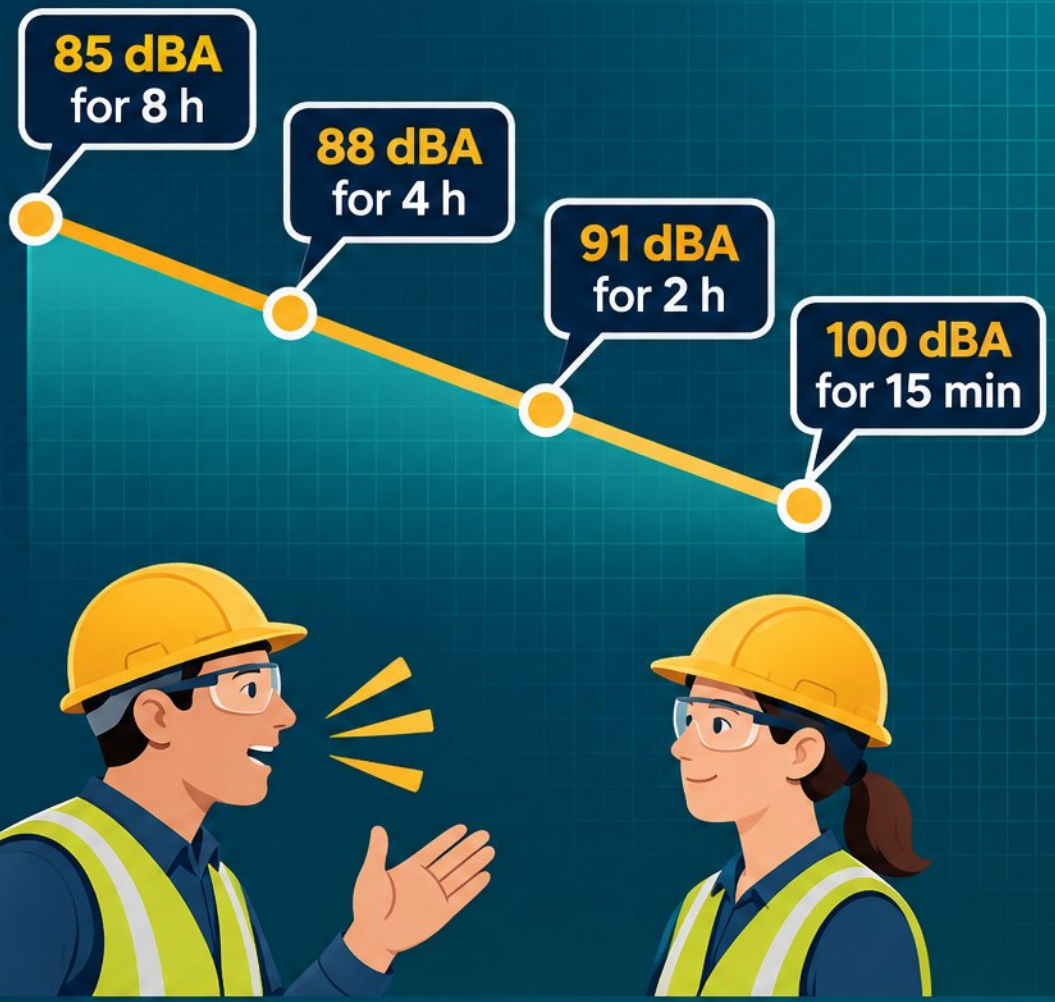
Scale Card

pH	dB	Magnitude



From Scale Reading to Real-World Decisions: Health and Safety

- NIOSH REL: **85 dBA** as an 8-hour time-weighted average.
- **3 dB** exchange rate: **+3 dBA** halves allowable exposure time.
- **85 dBA** for **8 h**; **88** for **4 h**; **91** for **2 h**; **100** for **15 min**.
- OSHA PEL: **90 dBA** for **8 h**, using a **5 dB** exchange rate.
- dBA is weighted to match hearing — the right unit for risk.
- If you must raise your voice at arm's length, noise likely exceeds **85 dBA**.



×10 ×10

From Scale Reading to Real-World Decisions: Environment and Preparedness









- Drinking water: EPA guideline pH 6.5–8.5; outside, pipes corrode or water tastes bitter
- Pools: pH 7.2–7.8; low pH irritates eyes and corrodes equipment
- Garden soil: crops prefer pH 6.0–7.0 for nutrient availability
- Freshwater life: EPA criteria pH 6.5–9; optimal near 6.5–8
- M6 vs M8: about 1,000× more energy, not a "slightly bigger" quake
- **Ask: relative to what reference? How many steps apart?**

Putting All Three Scales Side by Side

Scale	Key formula (specific form)	Unified template $L = c \cdot \log(Q/Q_0)$	c
 **pH**	$\mathrm{pH} = -\log[\mathrm{H}^+]$	$L = -1 \cdot \log(Q/Q_0)$	**−1**
 **Decibels**	$L = 10 \cdot \log(I/I_0)$, $I_0 = 10^{-12}\ \mathrm{W/m^2}$	$L = 10 \cdot \log(Q/Q_0)$	**10**
 **Magnitude**	$M = \log(A/A_0)$	$L = 1 \cdot \log(Q/Q_0)$	**1**

- **pH:** pH = −log[H⁺]; one unit lower = ×10 [H⁺]
- **Decibels:** $L = 10 \cdot \log(I/I_0)$, $I_0 = 10^{-12}$ W/m²; +10 dB = ×10
- **Magnitude:** $M = \log(A/A_0)$; one step = ×10 amplitude, ≈ ×31.6 energy
- **Unified template:** $L = c \cdot \log(Q/Q_0)$; c = −1 (pH), 10 (dB), 1 (magnitude)
- **Reading strategy:** check coefficient and sign, subtract, then exponentiate



Teaching and Learning Checkpoints

Quick check 1:

pH 2 vs pH 5 → 3 steps →
1,000× more acidic

Quick check 2:

80 dB vs 60 dB → 20 dB →
100× the intensity

Quick check 3:

M6 vs M8 → 100× amplitude,
~1,000× energy



Distractors to plant:

- "three times"
- "10^20"
- "pH 5 is more acidic"

Activities:

- paper log strips
- phone sound-level app
- energy calculator

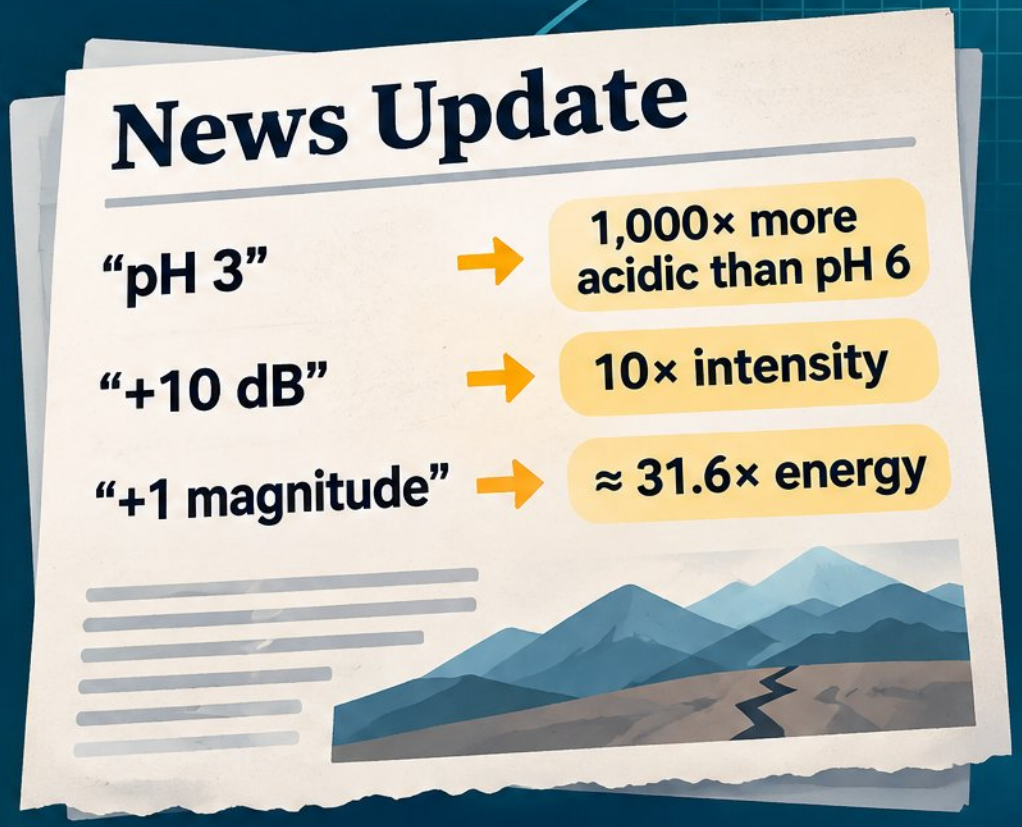
Reference card:

- three formulas
- three reference values
- "subtract, then power"

Wrap-Up: From 'How Much More?' to 'How Many Times More?'

- Log scales turn multiplication into addition: a scale difference is a ratio.
- Killer facts: pH 3 is 1,000× more acidic than pH 6; +10 dB = 10× intensity; +1 magnitude ≈ 31.6× energy.
- Use it for hearing safety, pools and soil, and reading earthquake news.
- Daily practice: convert one pH, one dB, and one magnitude comparison for a week.
- Teachers: reuse the paper log strips and card sort as a 20-minute activity.
- Closing prompt: rewrite a recent news pH, dB, or magnitude as a ratio.





Subtract-then-Power

Subtract → Then power → Get the ratio

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